Modelling the tank emptying

This paper describes a mathematical model of the emptying of a tank of saturated nitrous oxide that empties purely due to its self pressure. This can be called a 'blowdown' emptying.

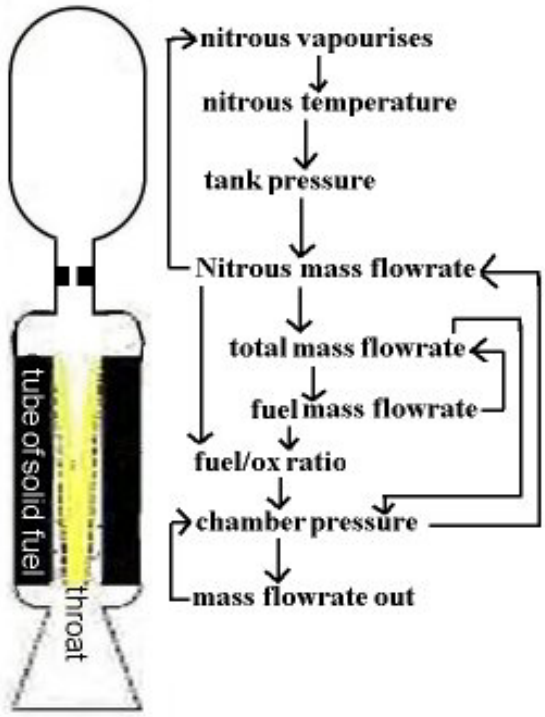
Firstly, read our 'The physics of nitrous oxide' article, pages 7 and 8 as it covers the processes we'll be modelling herein.

The mathematical model is based on a model of saturated propane emptying from a tank devised by Dr Bruce Dunn (ref. 1). Aspire gratefully acknowledges the help we received from Dr Dunn in the preparation of our nitrous tank model.

The nitrous hybrid as a system

The emptying process is iterative with time, and it's strongly coupled to the combustion chamber pressure.
So the model of the tank emptying has to be coupled to a simulation of the combustion chamber and nozzle throat of the hybrid rocket motor that it is feeding:

The feedback loops of a nitrous hybrid are as follows:



Nitrous flows out of the tank causing a drop in the level of liquid nitrous. This causes an increase in the head space of nitrous vapour above the liquid.
The nitrous vapour pressure drops due to this expansion.
Some of the liquid nitrous then vapourises to try to raise the vapour pressure back up.
The energy required to vapourise the liquid comes from the liquid itself, and so its temperature drops.
This lower temperature lowers the tank pressure.

The flowrate of nitrous out of the tank depends on the difference between the tank and combustion chamber pressure.

The fuel mass flowrate eroded from the plastic fuel grain depends on the total flowrate of fuel plus nitrous oxidiser.

The combustion chamber pressure depends on the fuel to oxidiser ratio, the gain of mass in the chamber (fuel flowrate + ox flowrate – flowrate out the nozzle), and the nozzle throat area.

The tank emptying

When the nitrous vapour expands due to the level of the liquid dropping, the pressure drops due to this expansion.
We don't need to know what this pressure drop is to model the tank emptying, instead we estimate how much mass of nitrous liquid, m_v, has been vapourised to try and raise the pressure back up again to as it was.

This is an iterative process; we pick an arbitrary nonzero value for m_v to start with, and the program quickly converges on the actual value, and stays with it as it changes as the tank empties.

We calculate the heat removed from the liquid nitrous during its vaporisation:
$\Delta Q = m_v H_v$ where H_v is the enthalpy (latent heat) of vaporisation evaluated at the current nitrous temperature.

We then calculate the temperature drop of the remaining liquid nitrous (m_{liquid}) due to losing this heat:
$\Delta T = \frac{-\Delta Q}{m_{liquid} C_{liquid}}$ where C_{liquid} is the Specific heat capacity of liquid nitrous at the current temperature.

We then subtract this temperature drop from the current liquid nitrous temperature to get a new lower liquid nitrous temperature.

The liquid density ρ_{liquid} , the vapour density ρ_{vapour} , and the vapour pressure (tank pressure) are now recalculated based on this lower temperature.

Using this new tank pressure and the current combustion chamber pressure, the mass flowrate of liquid nitrous out of the tank, $\dot{m}_{liquid}$, is now calculated:

Starting with Bernoulii's equation for the flow of nitrous from the injector manifold into the injector orifices:
$P_{manifold} + \frac{1}{2}\rho_{liquid} V_{manifold}^2 = P_{injector} + \frac{1}{2}\rho_{liquid} V_{injector}^2$

As the nitrous leaves the injector orifices, it breaks into droplets without changing pressure, so $P_{injector}$ = combustion chamber pressure.
The injector pressure drop can be checked to ensure that it is greater than 20% of the combustion chamber pressure for safety as advised in ref. 3

Substituting for the velocity of the liquid from a rearrangement of the mass continuity equation:
$V = \frac{\dot{m}_{liquid}}{\rho_{liquid} A}$ where $\dot{m}_{liquid}$ is the mass flowrate of liquid, and *A* is the cross-sectional area of the manifold or injector orifice.

gives:
$P_{injector} - P_{manifold} = \Delta P = \frac{\dot{m}_{liquid}^2}{2\rho_{liquid}} \left(\frac{K}{\left(N A_{injector}\right)^2} - \frac{1}{A_{manifold}^2} \right)$ *N* is the number of orifices.

Note the inclusion of a loss coefficient *K*. This represents the loss of total pressure due to viscous losses/turbulence as the flow flows through the edges of the orifice.
As the static pressure of the nitrous liquid drops as it passes through the orifices, it begins to vapourise. This means that what flows through the injector is a foam of liquid and bubbles, and so traditional tables of loss coefficients or discharge coefficients don't work for this mixed fluid. You have to tailor this *K* coefficient until the time taken to empty the tank matches your test results. We've found that a good starting value for *K* is 2.0 for nitrous.

Rearranging, and assuming that $A_{manifold}^2$ is much larger than the injector orifice area gives:

$$\dot{m}_{liquid} = \sqrt{\frac{2\rho_{liquid}\Delta P}{D_{loss}}} \text{ where } D_{loss} = \frac{K}{\left(N A_{injector}\right)^2}$$

Having got the mass flowrate of liquid out of the tank we can now integrate to get the mass that has left the tank in this time iteration:

Total system mass m_{total} (liquid + vapour) has decreased by $\dot{m}_{liquid}\Delta t$

Liquid mass m_{liquid} has decreased by $\dot{m}_{liquid}\Delta t$ The resulting value for m_{liquid} is the mass of liquid that would be in the tank if the nitrous did not react to the expansion of the nitrous vapour and the ensuing drop in pressure. We'll designate this as m_{liquid_old}

But the nitrous *does* react, both to the increase in nitrous volume and also the drop in temperature.

The densities of the liquid and vapour are functions of temperature only.
The nitrous is constrained to fit into the volume of the tank, so is forced to adhere to a volume formula:

$V_{vapour} + V_{liquid} = V_{tank}$ or, $\frac{m_{liquid}}{\rho_{liquid}} + \frac{m_{vapour}}{\rho_{vapour}} = V_{\tan k}$ where $m_{total} = m_{liquid} + m_{vapour}$

Rearranging:

$$m_{liquid} = \frac{\left(V_{\tan k} - \frac{m_{total}}{\rho_{vapour}}\right)}{\left(\frac{1}{\rho_{liquid}} - \frac{1}{\rho_{vapour}}\right)}$$ we'll designate this value as m_{liquid_new}

The discrepancy between this value and the previous value is the mass of nitrous that has been vapourised:

$m_v = m_{liquid\_old} - m_{liquid\_new}$

With this new value for m_v we can proceed to the next time iteration, and begin the calculation loop again.

Software
Some software extracts are listed in a separate Word document 'Tank emptying code.doc' comprising of a suite of subroutines that give the nitrous properties as functions of temperature etc called nitrous_oxide.cpp and its header file nitrous_oxide.h

Also listed are subroutines coding the tank emptying model as described above.

References:

Ref. 1: Dr Bruce P. Dunn
University of British Columbia and Dunn Engineering
Several articles on self pressurised peroxide rockets and experiments on propane tanks, as well as email communications with the author on the subject of numerical modelling of the tank emptying process; many thanks.

Ref. 2: Engineering Sciences Data Unit (ESDU) sheet 91022,
Thermophysical properties of nitrous oxide.
Available in hardcopy from some U.K. University libraries, or accessible over the Web to students with an ATHENS password.

Ref. 3: Space Propulsion Analysis and Design
by Ronald .W. Humble, Gary .N. Henry and Wiley J. Larson
McGraw Hill Space Technology Series ISBN 0-07-031320-2